The Two Couriers Problem

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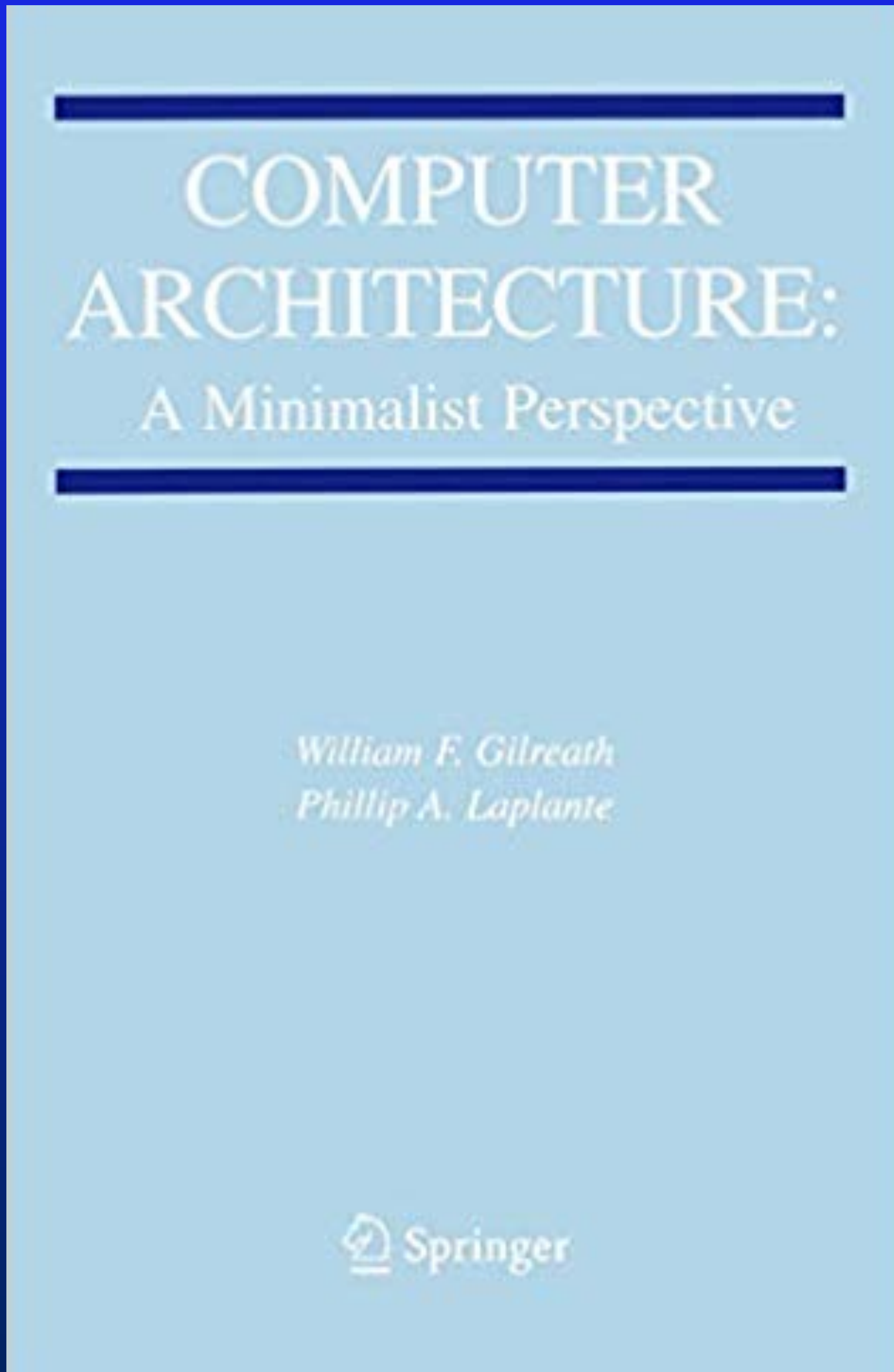
Hello! I am William Gilreath

- Author of the research paper
- Software development engineer, computer scientist, mathematician, writer
- https://wgilreath.github.io/WillHome.html

Some of my Works…

- “Division by Zero Paradoxes in Transmathematics” published by the General Science Journal October 2016
- Author of “Computer Architecture: A Minimalist Perspective” explores one-instruction set computing
- Author of “Non-Negative in Value but Absolute in Function—the Cogent Value Function” examines a new definition to the absolute value function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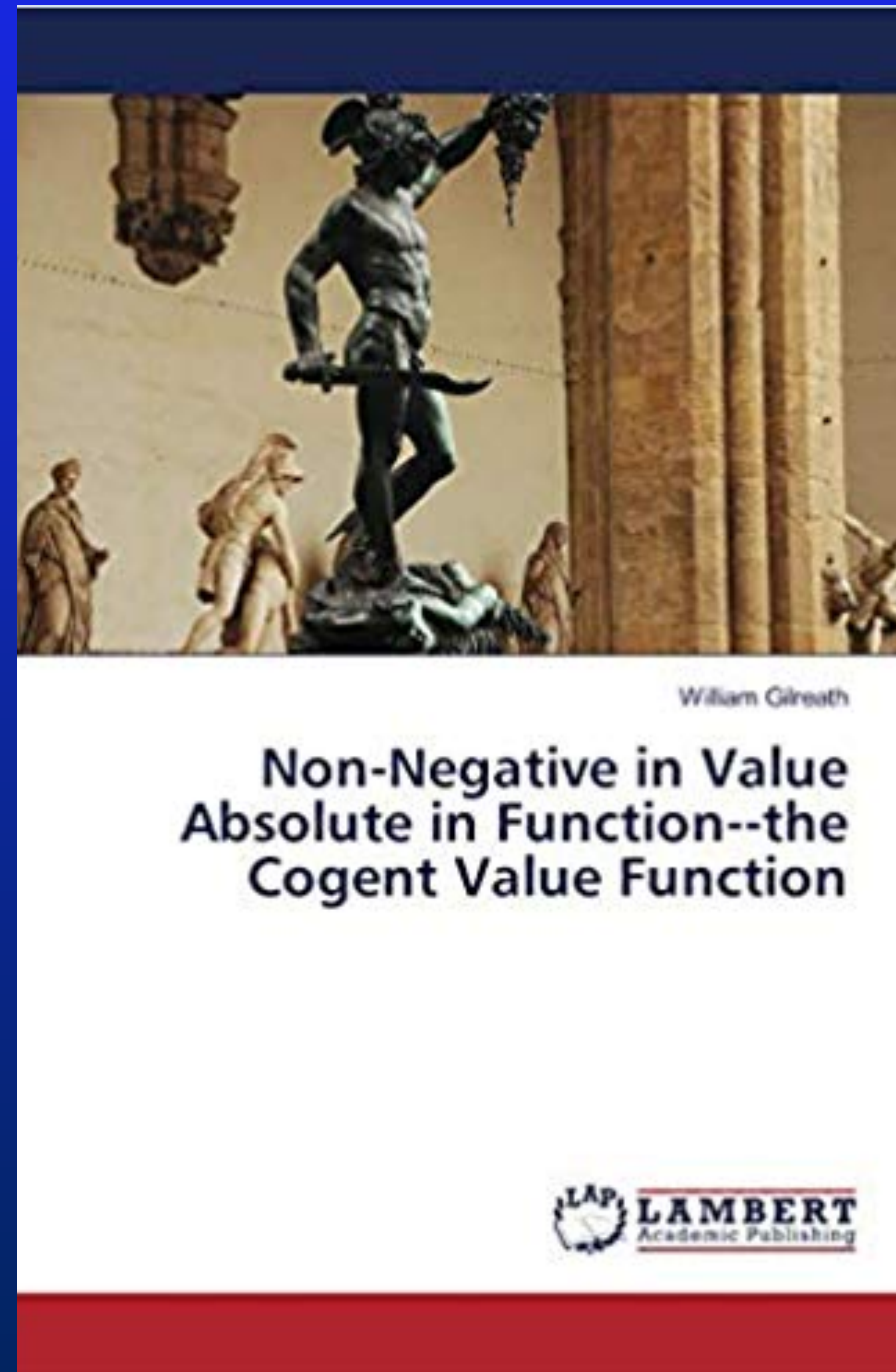
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Non-Negative in Value
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Presentation Approach:

- Significance to Transmathematics
- The History and Definition of the “Two Couriers Problem”
- Comparing Transmathematics to Conventional and Other Division by Zero Systems
- Conclusion

Significance to Transmathematics

What does a classic algebra problem have to do with transmathematics?

Division by Zero

Division by Zero—the Two Couriers Problem is an application in algebra that has division by zero

Means to Distinguish Other Systems of Division by Zero

How does conventional mathematics, and two other systems of division by zero solve the Two Couriers Problem?

Real World Application of Transreal Numbers

Nullity ϕ

Infinity ∞

The Problem - History and Definition

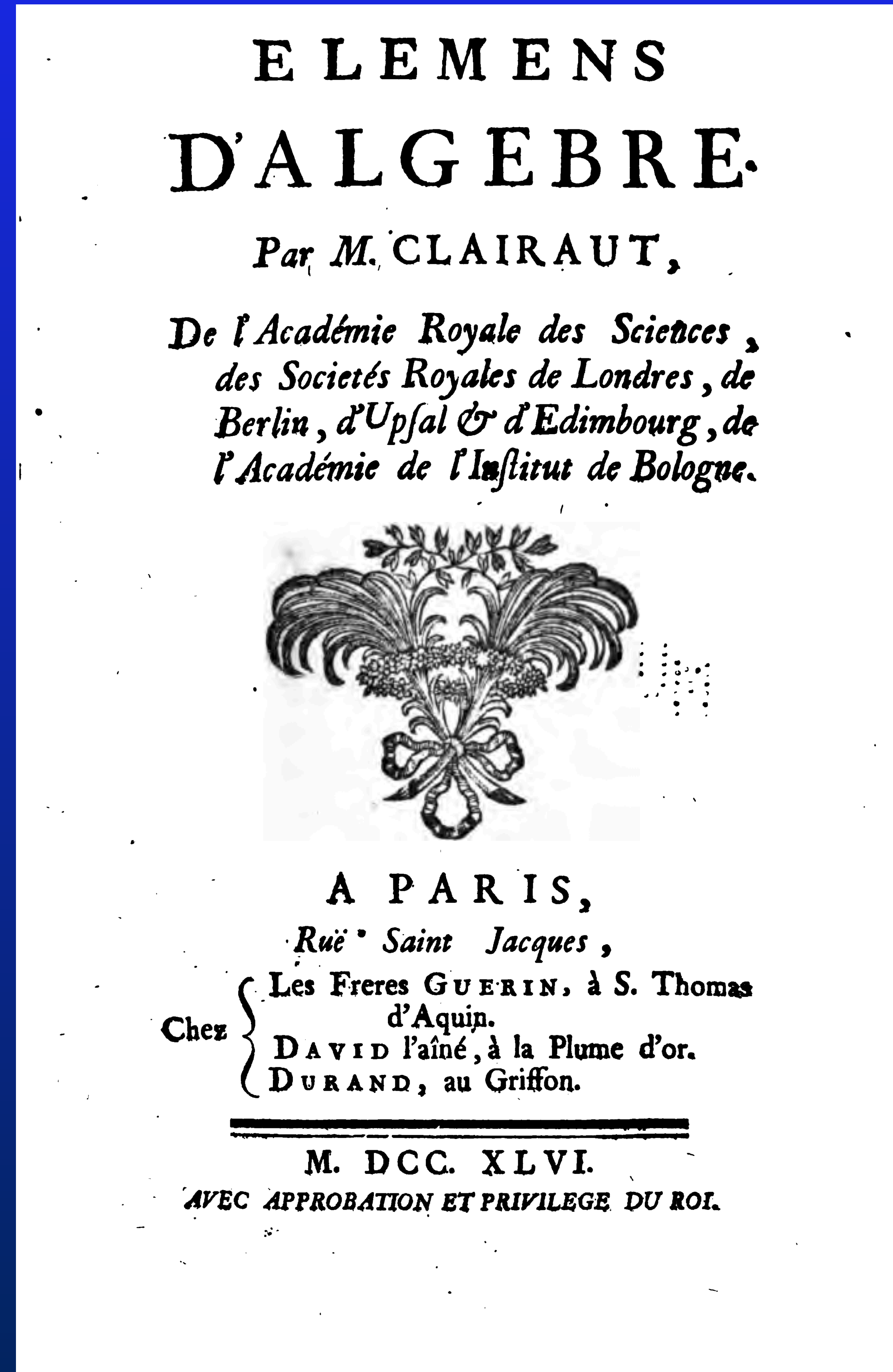
History

The Two Couriers Problem is 273-year old applied algebra problem!

Alexis Claude Clairaut (1713 – 1765)

- French mathematician, astronomer, geo-physicist
- Clairaut's Theorem: a mathematical law giving the surface gravity on a viscous rotating ellipsoid in equilibrium under the action of its gravitational field and centrifugal force
- Discovered approximate solution to three body problem in 1750 on how the Earth, moon, and Sun are attracted to one another

Source of the Two Couriers Problem: originates from **Elemens D'Algebre** 1746



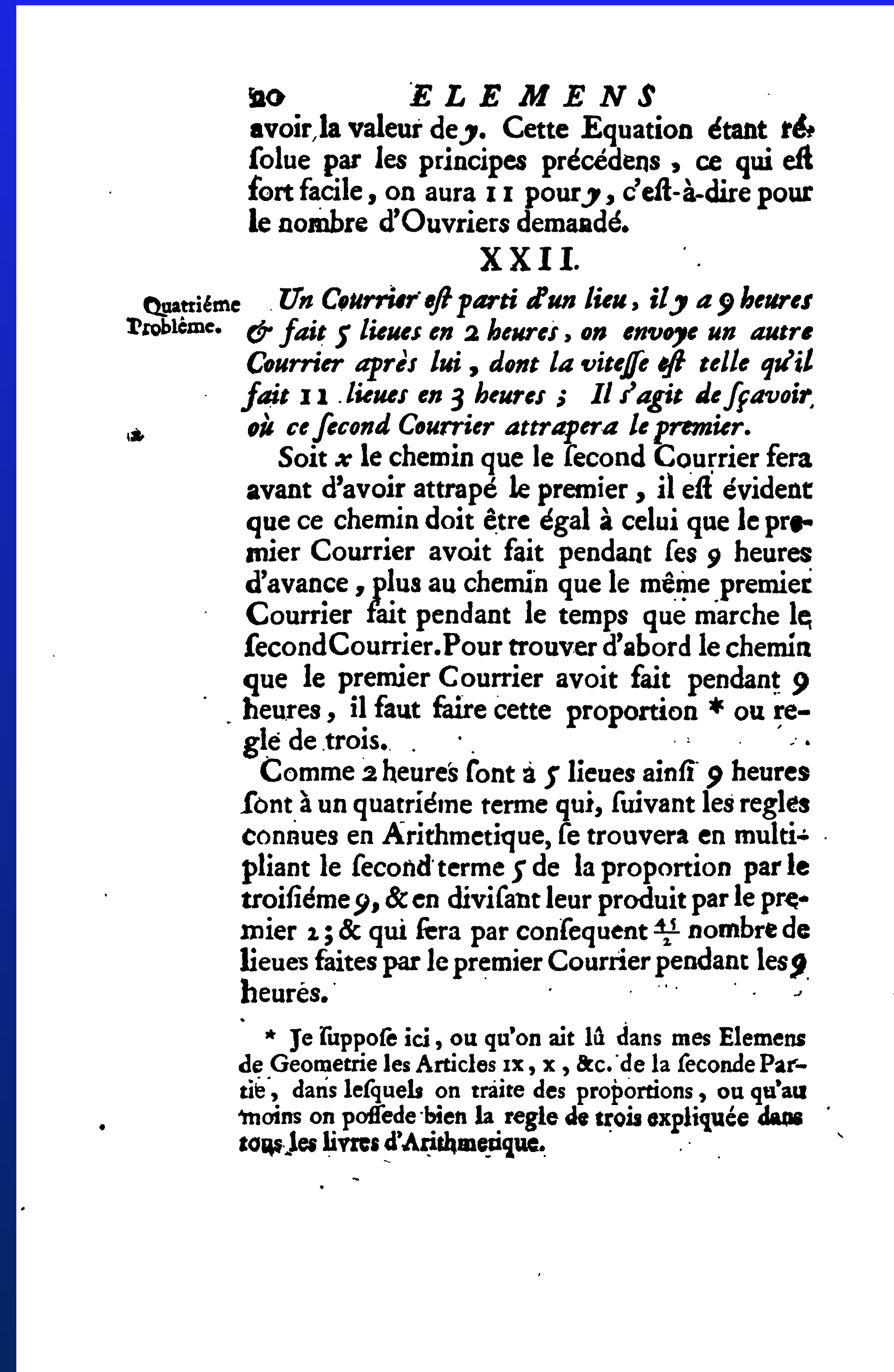
ELEMENS
D'ALGEBRE.
Par M. CLAIRAUT,
De l'Académie Royale des Sciences, des Societés Royales de Londres, de Berlin, d'Upsal & d'Edimbourg, de l'Académie de l'Institut de Bologne.

A PARIS,
Ruë Saint Jacques,
Chez { Les Freres GUERIN, à S. Thomas d'Aquin.
DAVID l'aîné, à la Plume d'or.
DURAND, au Griffon.

M. DCC. XLVI.
AVEC APPROBATION ET PRIVILEGE DU ROI.

Original problem in archaic decrepitude

Excerpt from p. 20 of Elemens D'Algebre



avoir la valeur de y. Cette Equation étant réſolue par les principes précédens, ce qui eſt fort facile, on aura 11 pour y, c'eſt-à-dire pour le nombre d'Ouvriers demandé.

XXII.

Quatriéme Probléme. *Un Courrier eſt parti d'un lieu, il y a 9 heures & fait 5 lieues en 2 heures, on envoye un autre Courrier après lui, dont la viteſſe eſt telle qu'il fait 11 lieues en 3 heures; Il s'agit de ſçavoir où ce ſecond Courrier attrapera le premier.*

Soit x le chemin que le ſecond Courrier fera avant d'avoir attrapé le premier, il eſt évident que ce chemin doit être égal à celui que le premier Courrier avoit fait pendant ſes 9 heures d'avance, plus au chemin que le même premier Courrier fait pendant le temps que marche le ſecond Courrier. Pour trouver d'abord le chemin que le premier Courrier avoit fait pendant 9 heures, il faut faire cette proportion * ou regle de trois.

Comme 2 heures ſont à 5 lieues ainſi 9 heures ſont à un quatriéme terme qui, ſuivant les regles connues en Arithmetique, ſe trouvera en multipliant le ſecond terme 5 de la proportion par le troiſiéme 9, & en diviſant leur produit par le premier 2; & qui ſera par conſequent $\frac{45}{2}$ nombre de lieues faites par le premier Courrier pendant les 9 heures.

* Je ſuppoſe ici, ou qu'on ait lû dans mes Elemens de Geometrie les Articles IX, X, &c. de la ſeconde Partie, dans leſquels on traite des proportions, ou qu'au moins on poſſede bien la regle de trois expliquée dans tous les livres d'Arithmetique.

Original Problem

- The formulation of the original problem is difficult to follow
- The problem has been restated in numerous textbooks onward over the centuries
- The last use of the problem the author found was in 1937 by Grover Cleveland Bartoo in **First-year Algebra: A Text-workbook**, Webster Publishing Company, St. Louis, Missouri, USA
- Best definition given by De Morgan

Augustus De Morgan (1806 - 1871)

- British mathematician and logician
- Gave the best formulation of the Two Couriers Problem
- Used the problem in **On the Study and Difficulties of Mathematics**, Taylor and Walton, London, England, 1837, pp. 37-39

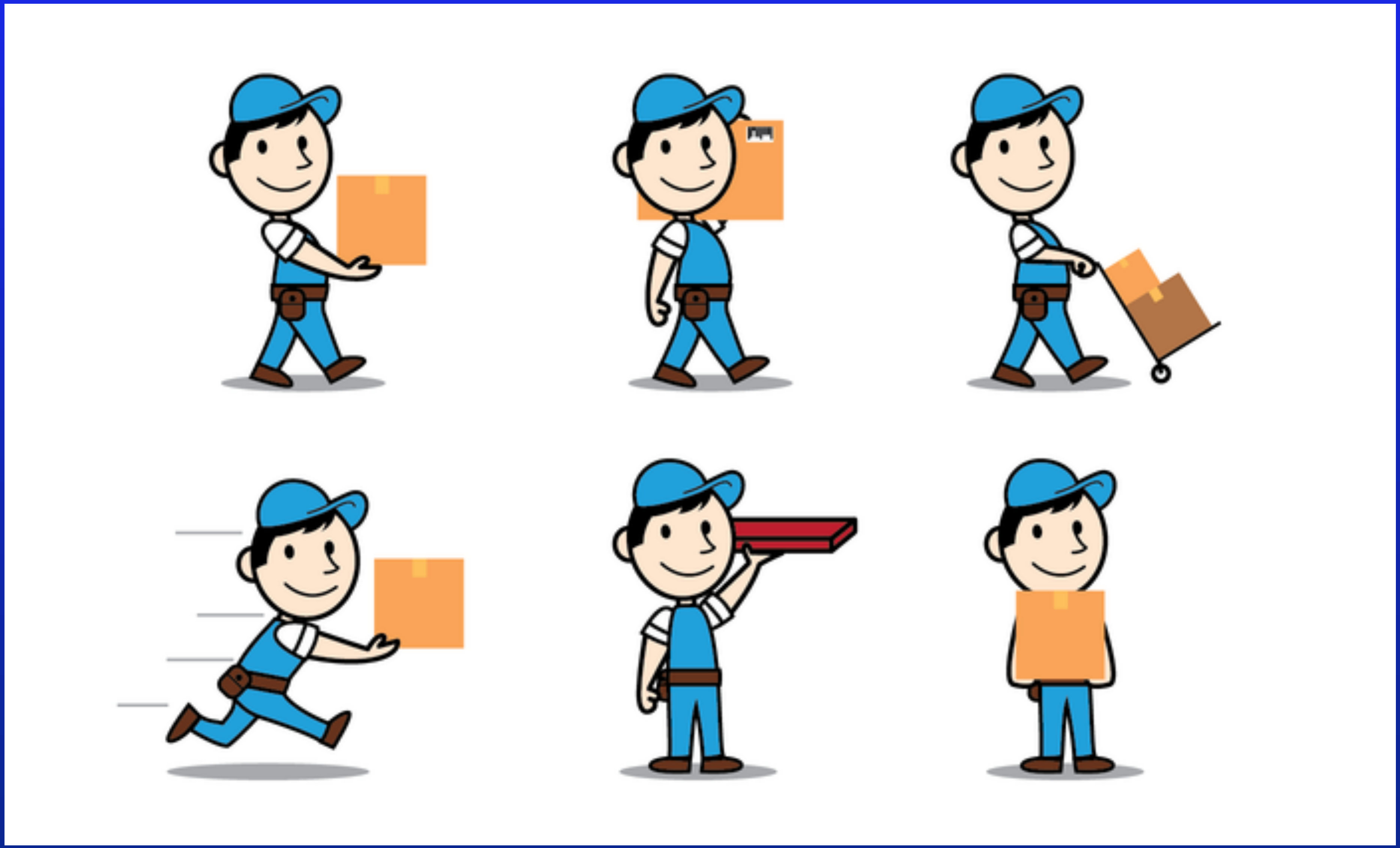
Definition

What is the problem?

“What we need then is not the right answer, but the right question,” Avon, from Blake’s 7 “Games”

De Morgan's Definition of the Problem…

“Two couriers, A and B, in the course of a journey between towns C and D, are the same moment of time at A and B. A goes m miles, and B, n miles an hour. At what point between C and D are they together?…Let the distance AB be called a.”



Six Cases to the Problem

"It is evident that the answer depends upon whether they are going in the same or opposite directions, where A goes faster or slower than B, and so on. But all these, as we shall see, are include in the same general problem..." (De Morgan)

Only Four Significant Cases

- The first four cases are simplified to an expression
- The time the two couriers will meet (or rendezvous?) is the distance between them
- The expression: a/(m-n) or a/(n-m)
- Note a is the distance between courier A, travelling at m miles per hour, and courier B, travelling at n miles per hour

Simplify further into two cases

- When $a > 0$ and $m = n$ is the case of $(a / 0)$
- When $a = 0$ and $m = n$ is the case of $(0 / 0)$
- Using transreal numbers, these are infinity and nullity

What does it mean for infinity?

- For (a/0) infinity it is the case there is always some distance a between couriers A and B.
- The couriers have the same speed m = n.
- Thus the two couriers will never meet, the point of rendezvous is the transreal infinity

What does it mean for nullity?

- For (0/0) nullity it is the case there is always no distance a = 0 between couriers A and B
- The couriers have the same speed m = n
- Thus the two couriers are together **always**, the point of rendezvous is at every point or the transreal nullity.

Nullity

$$\phi$$

Basically all points along the number line are a solution

Infinity

∞

There is no point where the two couriers meet

Other Systems for Division by Zero

- Conventional Mathematics
- Saitoh
- Barukčić
- Note there are other systems of division by zero so this is not an exhaustive comparison

Conventional Mathematics
0/0 = Indeterminate

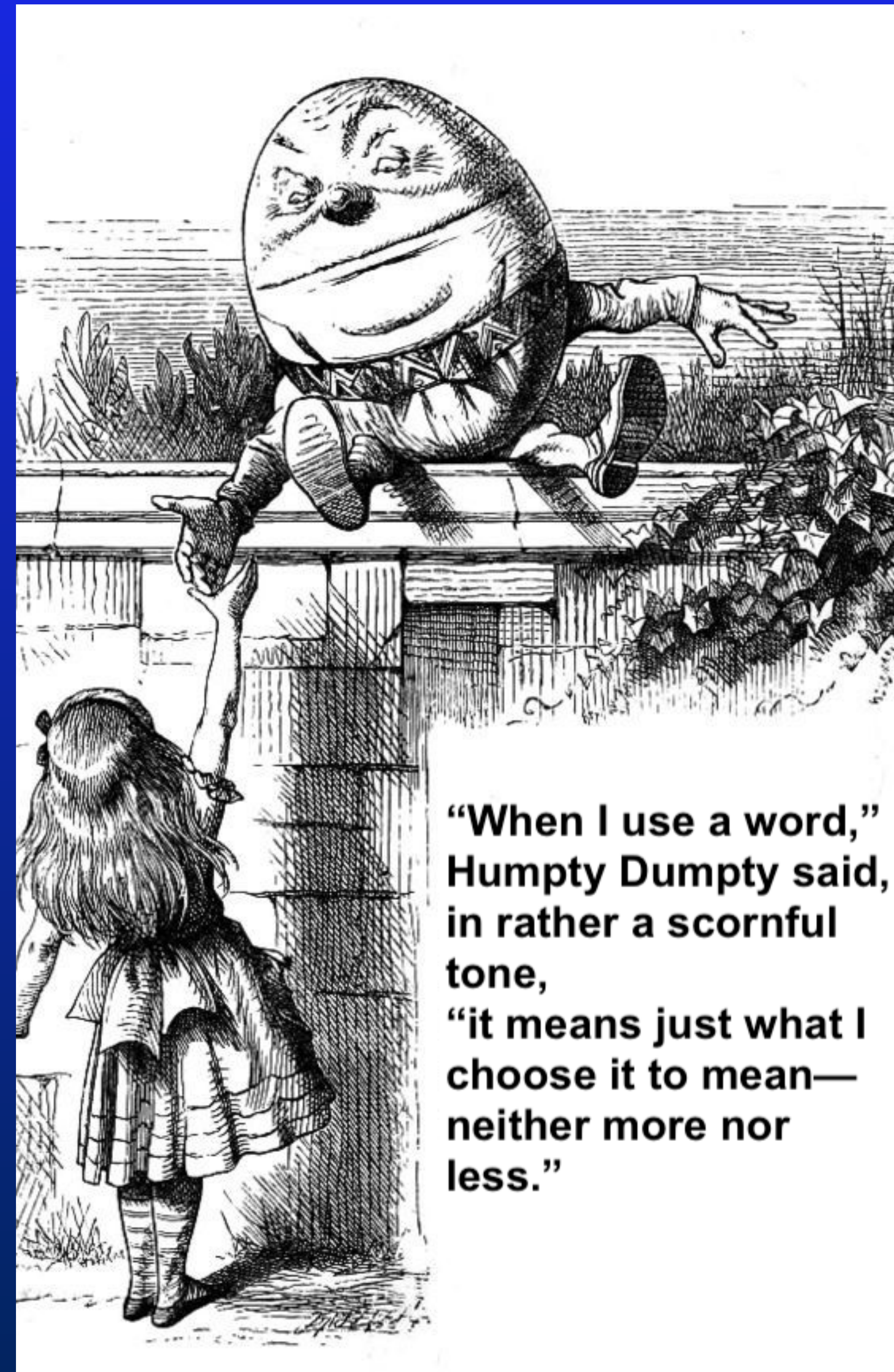
The use of the word ‘indeterminate’ is evasive and ambiguous

Math texts will use other terms like “undefined” or “unknown”

Conventional Mathematics Solution to Division by Zero

Words that are not a solution to division by zero

Lewis Carroll (1832–98) Through the Looking-Glass, Chapter 6, p. 205, 1934



- How “indeterminate” is indeterminate?
- Conventional mathematics gives us an answer that means three things:
  - Indeterminate
  - Undefined
  - Unknown
- Not very helpful since mathematics is about finding a solution with meaning

Saitoh, Barukčić

Both Saburo Saitoh and Ilija Barukčić
formally define division by zero, but differently

Saitoh

- Saitoh defines z/0 = 0 where z is any real number.
- Thus 0/0 = 0, n/0 = 0 where n != 0.
- There is no infinity in Saitoh's system for division by zero.

Saitoh and the Two Couriers Problem

- Saitoh's solutions to the two cases are 0 and 0
- The case of 0/0, the two couriers are always together, but 0/0 = 0
- The case of n/0, where n != 0, the two couriers never meet, but n/0 = 0

Saitoh’s Division by Zero System

Saitoh’s system can be summarized by a song lyric:

“Nothin' from nothin' leaves nothin'...”

”Nothing From Nothing” 1974 song by Billy Preston and Bruce Fisher

Barukčić

- Barukčić defines 0/0 = 1
- Any other division by zero is still conventional, so n/0 = infinity for n > 0
- Barukčić uses Einstein’s relativity theory as the basis for his definition

Barukčić's Solution to the Two Couriers Problem

- Barukčić's solutions are 1, and infinity
- The case of 0/0 the two couriers are always together, but 0/0 = 1
- The case of n/0 where n != 0 = infinity.

Barukčić's System of Division by Zero

Barukčić's system can be summarized with the old cliché pun:

"It's all relative."

Conclusion

Two Couriers Problem is nearly a Three Centuries old…

2019 - 1746 = 273

Yet, the best answer is indeterminate and infinite in conventional mathematics—without any real insight

Twenty-First Century Mathematics of Transmathematics Explains the Problem More Comprehensively

- Division by zero has a tangible transreal number as the result
- Two cases of division by zero have distinct transreal numbers
- Infinity for n/0 where n != 0
- Nullity for 0/0

Saitoh and Barukčić System Of Division by Zero

- Saitoh’s system is right for 0/0 = 0, but also wrong in that there are infinitely many other points

- Barukčić’s system is right for 0/0 = 1, but also wrong in that there are infinitely many other points

- Saitoh is wrong for n/0 = 0. The two couriers never meet

- Barukčić may be correct for n/0 = infinity; but he never clearly establishes what infinity is mathematically

Thus…

- Conventional mathematics is ambiguous, and ultimately that ambiguity is reflected in the heuristic “Do not divide by zero”
- Both Saitoh and Barukčić are partially correct in their respective systems
- Half a loaf is better than none, but it is not a comprehensive or general answer for division by zero

"Have You Divided by Zero, Lately?"

How Do you do it?

Transmathematics do It!