The Two Couriers Problem

William Gilreath
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Hello! I am William Gilreath

- Author of the research paper
- Software development engineer, computer scientist, mathematician, writer
- https://wgilreath.github.io/WillHome.html
Some of my Works…

• “Division by Zero Paradoxes in Transmathematics” published by the General Science Journal October 2016

• Author of “Computer Architecture: A Minimalist Perspective” explores one-instruction set computing

• Author of “Non-Negative in Value but Absolute in Function—the Cogent Value Function” examines a new definition to the absolute value function
Presentation Approach:

• Significance to Transmathematics

• The History and Definition of the “Two Couriers Problem”

• Comparing Transmathematics to Conventional and Other Division by Zero Systems

• Conclusion
Significance to Transmathematics

What does a classic algebra problem have to do with transmathematics?
Division by Zero—The Two Couriers Problem is an application in algebra that has division by zero.
Means to Distinguish Other Systems of Division by Zero

How does conventional mathematics, and two other systems of division by zero solve the Two Couriers Problem?
Real World Application of Transreal Numbers

Nullity $\phi$

Infinity $\infty$
The Problem -
History and Definition
History

The Two Couriers Problem is 273-year old applied algebra problem!
Alexis Claude Clairaut (1713 – 1765)

• French mathematician, astronomer, geo-physicist

• Clairaut's Theorem: a mathematical law giving the surface gravity on a viscous rotating ellipsoid in equilibrium under the action of its gravitational field and centrifugal force

• Discovered approximate solution to three body problem in 1750 on how the Earth, moon, and Sun are attracted to one another
Source of the Two Couriers Problem: originates from Elemens D’Algebre 1746
ELEMENS
avoir la valeur de $y$. Cette Equation étant résolue par les principes précédens, ce qui est fort facile, on aura $y$, c'est-à-dire pour le nombre d'Ouvriers demandé.

XXI.

Quatrième Problème.

Un Courrier est parti d'un lieu, il a $y$ heures et fait $5$ lieues en $2$ heures, on envoie un autre Courrier après lui, dont la vitesse est telle qu'il fait $11$ lieues en $3$ heures. Il s'agit de savoir où ce second Courrier attraperà le premier.

Soit $x$ le chemin que le second Courrier fera avant d'avoir attrapé le premier, il est évident que ce chemin doit être égal à celui que le premier Courrier ait fait pendant les $9$ heures d'avance, plus au chemin que le même premier Courrier fait pendant le temps que marche le second Courrier. Pour trouver d'abord le chemin que le premier Courrier ait fait pendant $9$ heures, il faut faire cette proportion $*$ ou règle de trois.

Comme $2$ heures font à $5$ lieues ainsi $9$ heures font à un quatrième terme qui, suivant les règles connues en Arithmétique, le trouvera en multipliant le second terme de la proportion par le troisième, et en divisant leur produit par le premier, et qui fera par conséquent la moitié de lieues faites par le premier Courrier pendant les $9$ heures.

*$*$ Je suppose ici, ou qu'on ait là dans mes Elemens de Geometrie les Articles $x$, et c. de la Seconde Partie, dans lesquels on traite des proportions, ou qu'autre chose posée bien la règle de trois expliquée dans tous mes livres d'Arithmetique.
Original Problem

• The formulation of the original problem is difficult to follow

• The problem has been restated in numerous textbooks onward over the centuries

• The last use of the problem the author found was in 1937 by Grover Cleveland Bartoo in First-year Algebra: A Text-workbook, Webster Publishing Company, St. Louis, Missouri, USA

• Best definition given by De Morgan
Augustus De Morgan
(1806 - 1871)

• British mathematician and logician

• Gave the best formulation of the Two Couriers Problem

Definition

What is the problem?

“What we need then is not the right answer, but the right question,” Avon, from Blake’s 7 “Games”
De Morgan's Definition of the Problem…

“Two couriers, A and B, in the course of a journey between towns C and D, are the same moment of time at A and B. A goes m miles, and B, n miles an hour. At what point between C and D are they together?…Let the distance AB be called a.”
Six Cases to the Problem

"It is evident that the answer depends upon whether they are going in the same or opposite directions, where A goes faster or slower than B, and so on. But all these, as we shall see, are include in the same general problem..." (De Morgan)
Only Four Significant Cases

• The first four cases are simplified to an expression

• The time the two couriers will meet (or rendezvous?) is the distance between them

• The expression: $a/(m-n)$ or $a/(n-m)$

• Note $a$ is the distance between courier A, travelling at $m$ miles per hour, and courier B, travelling at $n$ miles per hour
Simplify further into two cases

• When $a > 0$ and $m = n$ is the case of $(a / 0)$

• When $a = 0$ and $m = n$ is the case of $(0 / 0)$

• Using transreal numbers, these are infinity and nullity
What does it mean for infinity?

- For \((a/0)\) infinity it is the case there is always some distance \(a\) between couriers A and B.

- The couriers have the same speed \(m = n\).

- Thus the two couriers will never meet, the point of rendezvous is the transreal infinity.
What does it mean for nullity?

- For (0/0) nullity it is the case there is always no distance $a = 0$ between couriers A and B.
- The couriers have the same speed $m = n$.
- Thus the two couriers are together always, the point of rendezvous is at every point or the transreal nullity.
Nullity

ϕ

Basically all points along the number line are a solution
Infinity

∞

There is no point where the two couriers meet
Other Systems for Division by Zero

- Conventional Mathematics
- Saitoh
- Barukčić

Note there are other systems of division by zero so this is not an exhaustive comparison
Conventional Mathematics

$0/0 = \text{Indeterminate}$

The use of the word ‘indeterminate’ is evasive and ambiguous

Math texts will use other terms like “undefined” or “unknown”
Conventional Mathematics
Solution to Division by Zero

Words that are not a solution to division by zero

Lewis Carroll (1832–98)
Through the Looking-Glass, Chapter 6, p. 205, 1934

“When I use a word,” Humpty Dumpty said, in rather a scornful tone, “it means just what I choose it to mean—neither more nor less.”
• How “indeterminate” is indeterminate?

• Conventional mathematics gives us an answer that means three things:
  
  • Indeterminate
  
  • Undefined
  
  • Unknown

• Not very helpful since mathematics is about finding a solution with meaning
Both Saburo Saitoh and Ilija Barukčić formally define division by zero, but differently.
Saitoh

- Saitoh defines $z/0 = 0$ where $z$ is any real number.

- Thus $0/0 = 0$, $n/0 = 0$ where $n \neq 0$.

- There is no infinity in Saitoh’s system for division by zero.
Saitoh and the Two Couriers Problem

- Saitoh’s solutions to the two cases are 0 and 0
- The case of 0/0, the two couriers are always together, but 0/0 = 0
- The case of n/0, where n ≠ 0, the two couriers never meet, but n/0 = 0
Saitoh’s Division by Zero System

Saitoh’s system can be summarized by a song lyric:

“Nothin' from nothin' leaves nothin'...”

”Nothing From Nothing” 1974 song by Billy Preston and Bruce Fisher
Barukčić

- Barukčić defines $0/0 = 1$
- Any other division by zero is still conventional, so $n/0 = \infty$ for $n > 0$
- Barukčić uses Einstein’s relativity theory as the basis for his definition
Barukčić’s Solution to the Two Couriers Problem

• Barukčić’s solutions are 1, and infinity

• The case of 0/0 the two couriers are always together, but 0/0 = 1

• The case of n/0 where n != 0 = infinity.
Barukčić’s System of Division by Zero

Barukčić’s system can be summarized with the old cliché pun:

“It’s all relative.”
Conclusion
Two Couriers Problem is nearly a Three Centuries old…

2019 - 1746 = 273

Yet, the best answer is indeterminate and infinite in conventional mathematics—without any real insight
Twenty-First Century Mathematics of Transmathematics Explains the Problem More Comprehensively

- Division by zero has a tangible transreal number as the result
- Two cases of division by zero have distinct transreal numbers
- Infinity for $n/0$ where $n \neq 0$
- Nullity for $0/0$
Saitoh and Barukčić System Of Division by Zero

• Saitoh’s system is right for $0/0 = 0$, but also wrong in that there are infinitely many other points

• Barukčić’s system is right for $0/0 = 1$, but also wrong in that there are infinitely many other points

• Saitoh is wrong for $n/0 = 0$. The two couriers never meet

• Barukčić may be correct for $n/0 = \infty$; but he never clearly establishes what infinity is mathematically
Thus…

• Conventional mathematics is ambiguous, and ultimately that ambiguity is reflected in the heuristic “Do not divide by zero”

• Both Saitoh and Barukčić are partially correct in their respective systems

• Half a loaf is better than none, but it is not a comprehensive or general answer for division by zero
"Have You Divided by Zero, Lately?"

How Do you do it?

Transmathematics do it!